A Self Study Guide for

Dynamics in Engineering Mechanics

(For Under-Graduate Engineering Students)

by

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PREFACE

I must thank CMD of MADE EASY Group, **Mr. B. Singh** for providing me an opportunity to reach out to the Student Community at large through my present book **"A Self Study Guide for Dynamics in Engineering Mechanics"**. Students may be benefitted from my 60 years of teaching, research and publications through this book.

This book is an initiative to help the needy and mediocre students to self study the subject of **Dynamics** at home and build their concepts and prepare for examinations with confidence.

Questions in the book have been designed on the pattern of questions that are being asked in university examinations and competitive examinations of UPSC/GATE/PSUs.

The book has been thoroughly reviewed and questions from competitive examinations for the last two years have been added in the book.

Further improvements in the book will be made after getting the response from the students.

Any error in printing or calculations pointed out by the reader will be acknowledged with thanks by the author.

Dr. U. C. Jindal
Author

CONTENTS

Introduction to Dynamics

CHAPTER 1

Dynamics is that branch of Engineering Mechanics which deals with the study of rigid bodies in motion, i.e., study of velocity, acceleration and distance covered during the motion and the study of the forces causing the motion of rigid bodies such as traction effort during the motion of an automobile, thrust generated in jet propulsion, steam power required for a locomotive and power required for operaing a flywheel in a machine.

Kinematics deals with the study of motion and kinetics deals with the study of forces, thrust, moments, power during motion of rigid bodies.

Kinematics : In kinematics, we study the motion of a particle or rigid bodies such as motion of a rocket, motion of missile, crank-shaft mechanism, pendumum motion (harmonic motion) as shown in figures 1.1(a), (b), (c) and (d).

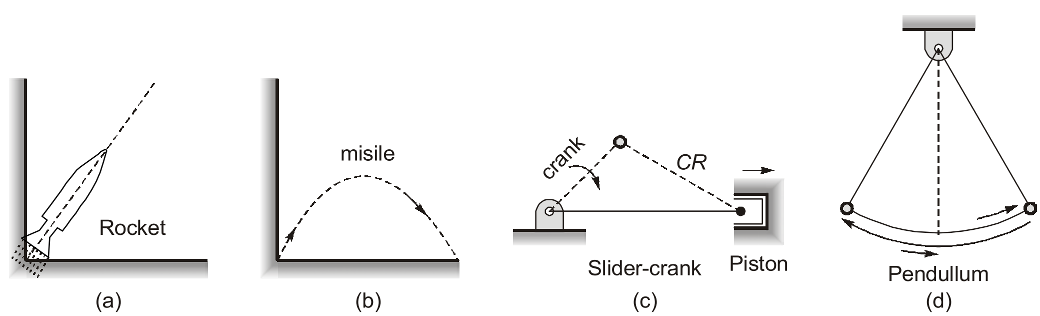


Figure : 1.1

During the study of motion of rigid bodies as missile, size and shape is of no relevance, study of velocity, acceleration and trajectory is made. The missile is considered as a particle.

1.1 Kinetics

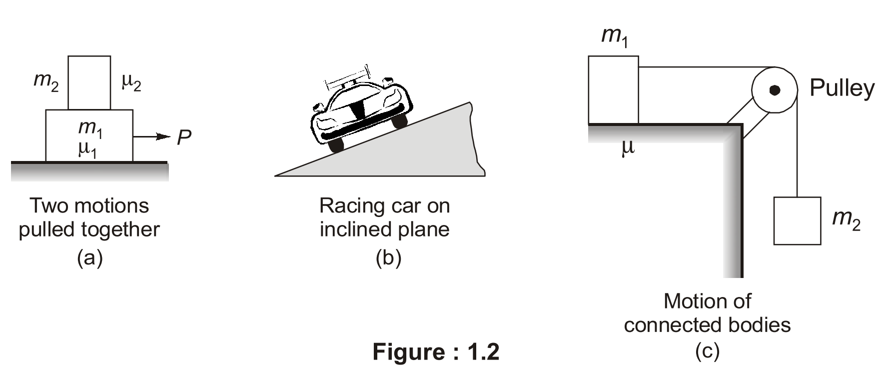


Figure : 1.2

In kinetics, we study about the effect of forces causing the motion of bodies such as (i) motion of two connected bodies under the action of a force P; (ii) Motion of a racing car along a curved path, creating centrifugal force on the car; (iii) motion of two connected bodies m_1 and m_2 through a cord, resulting in the acceleration of the bodies m_1 and m_2.

Galileo (1564–1642) made observations on the motion of falling bodies, motion of bodies on inclined plane, motion of pendulum and gave scientific approach to physical problem.

Huygen in 1657, Newton (1642–1727) formulated accurately the laws of motion. Law of universal gravitation was formulated by Newton.

Principles of dynamics help in the study of machines and structure moving with high speed and high acceleration.

1.2 Laws of Motion

Newton's three laws of motion, globally used by students and scientists can be briefly described as follows :

Law I : A particle continues to remain at rest or uniform motion (uniform velocity) unless it is acted upon by an external force.

Law II : The rate of change of momentum of a body is directly proportional to the applied force and the change of momentum takes place in the direction of applied force.

Law III : To every action, there is equal and opposite reaction, i.e., forces of action and reaction between interacting bodies are equal in magnitude and opposite in direction (at the same time collinear). Newton's second law of motion is the basis of Kinetic analysis.

Force, $$F = m \times a = \text{mass} \times \text{acceleration}$$

$$= m\left(\frac{v-u}{t}\right)$$

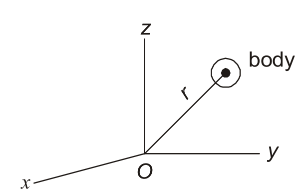
$$= \frac{\text{change of momentum}}{\text{time}}$$

where, $$\frac{v-u}{t} = \text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

1.3 Concepts of Dynamics

In dynamics, study is made on mass, time, force, vector, particle, rigid body, etc. during the motion.

Space : Space is a geometric region occupied by the body or bodies. In Newtonian Mechanics, position of a body in motion, is defined in an imaginary set of reference axis, x, y, z which remain fixed in space. This refeence frame x, y, z neither rotates, nor translates in space as shown in figure 1.3.



Primary inertial frame
(fixed in space)

Figure : 1.3

Say, initial position of body as defined by r.

$$r = xi + yj + zk$$

Motion can be defined in terms of linear or angular measurements, i.e., in terms of rectangular co-ordinates x, y, z or θ_x, θ_y, θ_z. (three dimensional spherical co-ordinate).

This frame is known as Primary Inertial Frame. Measurements of motion in the frame are known as absolute. These motions measurements are valid as long as velocity/acceleration of the body are negligible in comparison to the velocity of light.

A reference frame attached to the surface of the earth has somewhat complicated motion with reference to primary inertial frame, therefore, a correction factor is applied to the basic equation of motion in the case of rockets, space flight trajectories. Absolute motion of the earth becomes an important factor for the correction.

Mass : Mass is the amount of matter contained in a body. It is the measure of inertia or resistance to change in the motion of the body. The gravitational attraction of a body is proportional to the mass of the body.

Force : Force is the vector action of one body over another body and characteristic of force vector have been thoroughly discussed in the forthcoming chapters.

Time : This is the measure of succession of events and considered as an absolute quantity.

Particle : Particle is a body of negligible dimension when the motion of a body is described, the dimension or size of the body are irrelevant. A locomotive has a larger mass but when only the motion of the locomotive is described, the size of the locomotive and its dimension are not considered. Similarly, in the case of trajectory of a projectile, size of projectile is not taken into account.

Rigid Body : When forces are applied on a body, there are deformations in the body (slight change in dimension), these deformations are not taken into account and body is considered as rigid.

Vector and Scalar Quantities : Vector and scalar quantities have been thoroughly discussed in volume-I on **STATICS**.

1.4 Units

International system of metric units (or SI units) will be used in the book. There are four fundamentals quantities of units used in dynamics.

Quantity	Dimensional Symbol	SI Units	
		Unit	Symbol
Mass	M	Kilogram	kg
Length	L	Metre	m
Time	T	Second	s
Force	F	Newton	N

As the mass is taken as a base quantity, SI system is known as **absolute system**. In SI units, one Newton is that force, which **one kilogram mass has with acceleration of 1 m/s^2.**

1.5 Gravitation

Newton's law of gravitation gives the force of attraction between two bodies as :

$$F = \frac{Gm_1m_2}{r^2}$$

where, F = mutual force of attraction between two bodies of masses m_1, m_2

G = a universal constant called constant of gravitation

$= 6.673 \times 10^{-11}$ m^3/kg.s^2, obtained from experimental data

The only gravitational force of appreciable magnitude is the force due to attraction of the earth.

The force of mutual attraction between two bodies or force of gravitational attraction on a body due to earth is expressed in Newton (N) in SI units.

m_1 = mass of the body

m_2 = mass of the earth

R = radius of curvature of the earth

Force of attraction on a body due to earth

$$= m_1 g$$

$$= \frac{Gm_1m_e}{R^2}$$

Acceleration due to gravity, $g = \dfrac{Gm_e}{R^2}$

m_e = mass of earth

= 5.976×10^{24} kg through experimental observation

R = Radius of the earth

= 6.371×10^6 m

Putting these values, $g = \dfrac{6.673 \times 10^{-11} \times 5.976 \times 10^{24}}{(6.371 \times 10^6)^2}$

= 9.8246 m/s^2

The acceleration due to gravity has been determined considering xy reference axis system located at the centre of the earth, this centre does not rotate with the earth. With reference to these fixed axes (xyz at centre), absolute velocity of g is determined.

But the earth rotates and the acceleration of a freely falling body as determined from a position attached to the surface of the earth is slightly less than the absolute value.

Earth is rotating as an oblate spherical with flatterning at the poles. The standard value of g that has been adopted internationally relative to the rotating earth at sea level and a latitude of 45° is 9.8065 m/s^2.

In all engineering applications, in SI unit, g = 9.81 m/s^2 is taken at sea level.

Say, g_0 is the acceleration due to gravity at sea level and g is the acceleration due to gravity at an altitude h, the absolute value of g at an altitude h is given by :

$$g = g_o \times \frac{R^2}{(R^2 + h^2)}$$ where, R is the radius of the earth.

g on a body may be determined by a simple fact on a body falling is under gravity in a vacuum.

Say, m = mass of the body

g = absolute acceleration due to gravity

W = true value of weight of the body

$W = mg$

The approximate weight of a body is obtained by a spring attached to the surface of the earth is slightly less than true weight. The difference is due to rotation of the earth.

Question 1.1 Two steel balls diameter 50 mm each are touching each other. Determine the magnitude of the force of attraction between the balls. Density of steel = 7800 kg/m^3

Solution:

Diameter of each ball, d = 50 mm

Volume of each ball $= \dfrac{\pi}{6}(50)^3 = 65.45 \times 10^3$ mm^3

$= 65.45 \times 10^{-6}$ m^3

Mass of each ball, $m_1 = m_2 = 65.45 \times 10^{-6} \times 7800$ kg

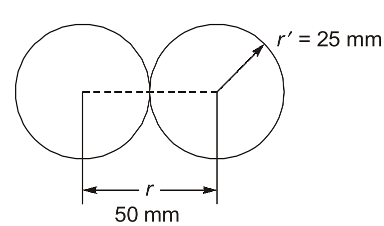


Figure : 1.4

$$= 510 \times 10^{-3} \text{ kg} = 0.51 \text{ kg}$$

Figure 1.4 shows two balls touch each other.

$$r = \text{distance between the centre}$$
$$= 50 \text{ mm} = 0.05 \text{ m}$$

Force of attraction between the balls

$$F = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.6373 \times 10^{-11} \times 0.51 \times 0.51}{0.05^2}$$
$$= 694.26 \times 10^{-11} \text{ N}$$
$$= 6.94 \times 10^{-9} \text{ N}$$
$$= 6.94 \text{ Nano Newton}$$

Practice Q.1.1 Two steel balls of diameter 40 mm each are touching each other. Determine the magnitude of the force of attraction between the bodies. Density of steel = 7800 kg/m^3, $G = 6.673 \times 10^{-11}$.

Ans. [2.8413 Nano Newton]

Question 1.2 Weight of a person on the surface of the earth is 700 N. At what height from the surface of the earth, the weight of the person will be 600 N?

Solution:

Say at height h, person's weight is reduced to 600 N. Radius of earth R.

g_o = acceleration due to gravity at the surface of the earth = 9.81 m/s^2

g = acceleration due to gravity at height h from the surface of the earth

$$= \frac{600}{700} \times g_o = 0.857 g_o$$

$$\frac{g}{g_o} = \frac{R^2}{(R+h)^2} = 0.857$$

$$R^2 = 0.857(R+h)^2$$

$$R = \sqrt{0.857}(R+h) = 0.9258(R+h)$$

$$h = \frac{R - 0.9258R}{0.9258} = \frac{0.0742}{0.9258} \times R$$

$= 0.080R$, where R = radius of the earth

Practice Q.1.2 Weight of a person on the surface of the earth is W. At what altitude above the north pole, the weight of the person is reduced to $0.2W$. Assume a spherical earth and radius of earth R. Determine h in terms of R.

Ans. [$1.236R$]

Question 1.3 Calculate the force of attraction exerted by the sun on a 70 kg man as he stands on the surface of the moon.

Solution:

G = Gravitational constant = 6.673×10^{-11} m^3/kg s^2

Mass of the earth, $m_e = 5.976 \times 10^{24}$ kg

Mass of the sun = $333000 m_e$

Distance between earth and sun = 149.6×10^6 km

Distance between earth and moon = 384398 km

$= 0.384398 \times 10^6$ km

Distance between sun and moon, $d_1 = 149.6 \times 10^6 - 0.384398 \times 10^6$ km

$= 149.265602 \times 10^9$ m

Mass of the sun = $333000 \times 5.976 \times 10^{24}$ kg

Mass of the man = 70 kg

Force exerted by sun on man standing on moon

$$F = \frac{Gm_1m_2}{d^2}$$

$$= \frac{6.673 \times 10^{-11} \times 70 \times 333000 \times 5.976 \times 10^{24}}{(149.205602 \times 10^9)^2}$$

$$= \frac{9.2955 \times 10^3}{22205.295} = 0.4175 \text{ N}$$

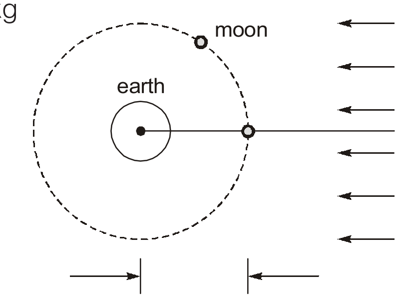


Figure : 1.5

Note that when the moon goes on the other side of earth, force exerted by sun on moon standing on moon is slightly reduced.

Practice Q.1.3 Calculate the force exerted by the moon on a 80 kg man as he stands on the surface of the moon. Given :

Mass of the moon = 0.0123 × mass of the earth

Radius of moon = 1738 km

Ans. [129.8 N]

Question 1.4 Determine the ratio of the force exerted by the sun on the moon and the force exerted by the earth on moon as shown in figure 1.6.

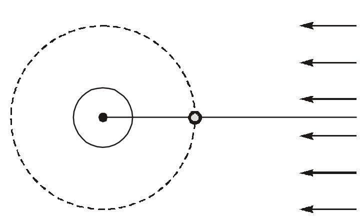


Figure : 1.6

Solution:

Distance between moon and earth = 384398 km = 0.384398×10^9 m

Mass of the moon = $m_m = 0.0123 m_e$

Mass of sun, $m_s = 333000 m_e$

Gravitational constant G.

Force exerted by earth on moon,

$$F_1 = \frac{Gm_m m_e}{(384398)^2}$$

$$= \frac{G \times 0.0123 m_e^2}{(0.384398 \times 10^9)^2}$$

$$F_1 = G \times 0.83242 \times 10^{-8} m_e^2$$

Distance between earth and sun = 149.6×10^6 km

Distance between moon and sun = 149.2156×10^6 km

= 149.2156×10^9 m

Force exerted by sun on moon,
$$F_2 = \frac{Gm_m m_s}{(149.2156 \times 10^9)^2}$$

$$= G \times \frac{33000 \times 0.0123 m_e^2}{22205.295 \times 10^{18}}$$

$$= G \times 0.18396 \times 10^{-18} m_e^2$$

Ratio,
$$\frac{F_2}{F_1} = \frac{0.18396}{0.083242} = 2.2099$$

$$= \frac{\text{Force exerted by sun on moon}}{\text{Force exerted b y earth on moon}}$$

Practice Q.1.4 Determine the ratio of the force exerted by the sun on the moon to that exerted by the earth on moon for position A of the moon.

Given : $G = 0.673 \times 10^{-11}$

m_e = mass of earth = 5.976×10^{24} kg

m_m = mass of moon = $0.0123 m_e$

m_s = mass of sun = $333000 m_e$

Centre to centre distance between moon and earth

= 384398 km

Earth to sun distance = 149.6×10^6 km

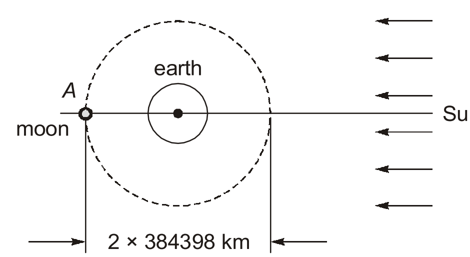


Figure : 1.7

Ans. [2.187]

■■■■

Important Points to Remember

1. In kinematics, motions of rigid bodies is studied such as displacement, velocity, acceleration in different types of motions as rectilinear, curvilinear motion, projectile motion, relative motion.
2. In kinetics, study of the effect of forces causing the motion is made as tractive force, centrifugal force etc.
3. Force = mass × acceleration = change of momentum per unit time
4. Space is a geometric region occupied by the body or bodies. In Newtonian Mechanic, position of a body in motion, is defined by an imaginary set of reference axis x, y, z which remain **fixed in space**.
5. Force is the vector action of one body over another body.
6. Units :

Mass	M	kg	Kilogram
Length	L	m	metre
Time	T	second	s
Force	F	Newton	N

7. Gravitation : Force of attraction between two bodies

$$F = \frac{Gm_1m_2}{r^2}$$

F = mutual force of attraction

G = a universal constant called constant of gravitation

= 6.673×10^{-11} m^3/kg s^2 obtained from experimental data

m_e = mass of earth = 5.976×10^{24} kg

R = radius of the earth = 6.371×10^6 m

g = acceleration due to gravity = 9.81 m/s^2

$$g = g_o \times \frac{R^2}{R^2 + h^2} \quad \text{where } R = \text{radius of the earth}$$

g_o = acceleration due to gravity at sea level

h = altitude at which g is determined

W = Weight = mg

8. Mass of sun, $m_s = 333000m_e$
9. Distance between earth and sun, $d = 149.6 \times 10^6$ km
10. Mass of moon, $m_m = 0.0123 \times m_e$ (mass of earth)

■■■■

Objective Type Questions

Q.1 Value of gravitational constant G is :

(a) 6.673×10^{-13} m^3/kg s^2

(b) 6.673×10^{-11} m^3/kg s^2

(c) 6.673×10^{-11} m^2/kg s^2

(d) None of these

Q.2 Radius of the earth is

(a) 6.371×10^3 km (b) 6.731×10^3 km

(c) 6.173×10^3 km (d) None of these

Q.3 Earth is considered as :

(a) perfectly spherical body

(b) a rotating oblate spheroid with flattening at the poles

(c) a rotating oblate spheroid

(d) All the above depending upon the situation

Q.4 A man is attracted by earth by 720 N force. If the man stands on the moon, the gravitational attraction on man from moon is 119 N. If gravitational attraction for earth is 9.81 m/s^2, what is the gravitational attraction for moon?

(a) 9.81 m/s^2 (b) 1.40 m/s^2

(c) 1.62 m/s^2 (d) None of these

Q.5 Maximum distance between moon and sun is

(a) 150.9844×10^6 km

(b) 149.2156×10^6 km

(c) 149.9844×10^6 km

(d) None of these

Answers

1. (b)	2. (a)	3. (d)	4. (c)	5. (c)

■■■■

Rectilinear Motion

CHAPTER 2

In rectilinear motion, particles of a body move along **parallel lines**. If a line is drawn on the body, then direction of this line remains the same throughout the motion. There are two types of motion of translation, i.e., (i) rectilinear; (ii) curvilinear translation as shown in figure 2.1(a) and (b).

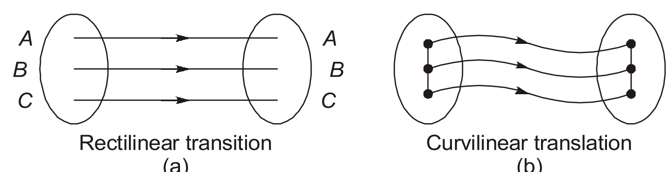


Figure : 2.1

In the case of **rectilinear motion**, all the particles of a body travel along straight parallel paths as shown in figure 2.1(a). In the case of curvilinear motion, particles of the body travel along curved path parallel to each other as shown in figure 2.1(b).

In this chapter we will study only the **rectilinear motion** as shown in figure 2.1(a).

Consider a body in x-y coordinate system, say line drawn on body is AB, as shown in figure 2.2. Position vectors of A and B are r_A and r_B respectively and vector $AB = r_{B/A}$.

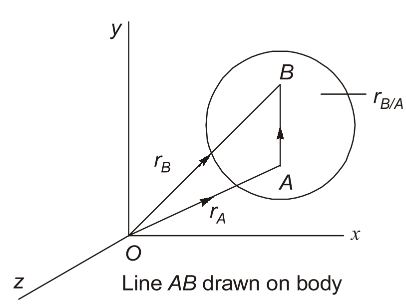


Figure : 2.2

Then, $$r_B = r_A + r_{B/A} \quad \text{...(1)}$$

Taking derivatives with respect to time of equation (1),

$$\frac{\partial r_B}{\partial t} = \frac{\partial r_A}{\partial t} + \frac{\partial r_{B/A}}{\partial t} \quad \text{...(2)}$$

Now in the motion of translation, line AB maintain its direction all along motion.

So, $$\frac{\partial r_{B/A}}{\partial t} = 0$$

so that $$\frac{\partial r_B}{\partial t} = \frac{\partial r_A}{\partial t} \quad \text{...(3)}$$

or velocity, v_B = velocity, v_A

Velocity of the particle B is equal to the velocity of the particle A.

Differentiating the equation (3) with respect to time, we get

$$\frac{\partial^2 r_B}{\partial t^2} = \frac{\partial^2 r_A}{\partial t^2} \quad \text{...(4)}$$

or $a_B = a_A$

Acceleration of the particle B = Acceleration of the particle A

Conclusion : When a rigid body is in rectilinear translation, all the points on the body have same velocity and same acceleration at any given instant. Moreover, all the particles of the body move along parallel straight line, while their velocity and acceleration keep the same direction throughout the motion.

Figure 2.3(a) and (b) shows *s*-*t* (displacement vs. time) curve and *v*-*t* (velocity vs. time) curves.

Variation of *s* with respect to time *s*-*t* curve and *v*-*t* curve variation of velocity with respect to time curve.

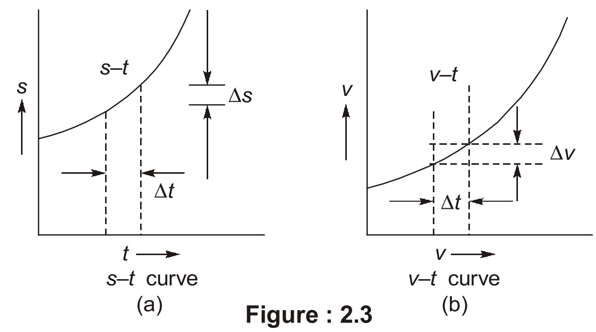


Figure : 2.3

Average velocity at any instant, $V_{av} = \dfrac{\Delta s}{\Delta t}$

In the limit, $$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad ...(5)$$

Similarly, figure 2.3(b) shows *V*-*t* curve.

At any instant, average acceleration,

$$a_{av} = \frac{\Delta V}{\Delta t}$$

In the limit, $$a = \lim_{\Delta t \to 0} \frac{dV}{dt} \qquad ...(6)$$

Different Cases in Rectilinear Motion : In rectilinear motion, acceleration may remain constant or may vary with respect to time, velocity and displacement. So, there are four different cases of rectilinear translation as follows :

(i) Acceleration is constant, such as motion of rigid bodies falling under gravity, acceleration due to gravity, *g* remain constant.

(ii) Acceleration is dependent on time, $a = f(t)$

(iii) Acceleration as a function of velocity, $a = f(v)$

(iv) Acceleration as a function of displacement, $a = f(s)$

2.1 Motion with Constant Acceleration

Acceleration, $a = \text{Constant}$

or $$\frac{dV}{dt} = a$$

or $$dV = a.dt$$

or $$\int dV = \int a dt$$

or $V = at + C_1$, where C_1 is constant of integration

If initial velocity is V_o at $t = 0$,. $V = V_o$

So, $$V = at + V_o \qquad ...(1)$$

Moreover, $$V = \frac{ds}{dt}$$

$$dS = V\,dt = (V_o + at)dt$$

$$S = V_o t + \frac{1}{2}at^2 + C_2 \qquad ...(2)$$

where, C_2 = another constant of integration

at time $t = 0$, $S = 0$, displacement is zero

So, constant $C_2 = 0$

Finally, $$S = V_o t + \frac{1}{2}at^2 \quad \text{...(3)}$$

From equation (1), $$t = \frac{V - V_o}{a}$$

Putting this value of t in equation (3), we get

$$S = V_o\left(\frac{V - V_o}{a}\right) + \frac{1}{2} \times a\left(\frac{V - V_o}{a}\right)^2$$

$$= \frac{V_o(V - V_o)}{a} + \frac{(V - V_o)^2}{2a}$$

or $$2aS = 2(V - V_o)V_o + (V - V_o)^2$$

$$= 2VV_o - 2V_o^2 + V^2 + V_o^2 - 2VV_o$$

$$2aS = V^2 - V_o^2$$

Following equations are equations for rectilinear motion, with constant acceleration :

1. $V = V_o + at$
2. $S = V_o t + \frac{1}{2}at^2$
3. $2aS = V^2 - V_o^2$

Question 2.1 For an aircraft, jet thrust remains constant or acceleration remains constant during take-off and can be taken equal to $\frac{g}{4}$. If the take-off speed is 180 km/hr, calculate the distance S and time t from rest to take-off of the aircraft.

Solution:

Acceleration of jet, $$a = \frac{g}{4} = \frac{9.81}{4} = 2.4525 \text{ m/s}^2$$

Take-off speed, $$V = 180 \text{ km/h} = \frac{180 \times 1000}{3600}$$

$$= 50 \text{ m/s}$$

Time taken to reach the speed, $$t = \frac{V}{a} = \frac{25}{2.4525} = 20.375 \text{ second}$$

Distance covered, $$S = \frac{1}{2}at^2 = \frac{1}{2} \times 2.4525 \times (20.375)^2$$

$$= 509 \text{ m}$$

Practice Q.2.1 For an aircraft, jet thrust remains constant or the acceleration is constant and can be taken equal to $\frac{g}{3}$. If the speed of take-off is 220 km/hour, calculate the distance S and the time taken from rest to take-off of the aircraft.

Ans. [18.64 second, 571 m]

Question 2.2 A particle moves along x-axis with an initial velocity of 24 m/s. The velocity remain constant for first 5 second, thereafter an acceleration of –4 m/s^2 acts on the particle. Determine when the velocity of the particle becomes zero and the distance covered by the particle in 15 seconds. (figure 2.4).

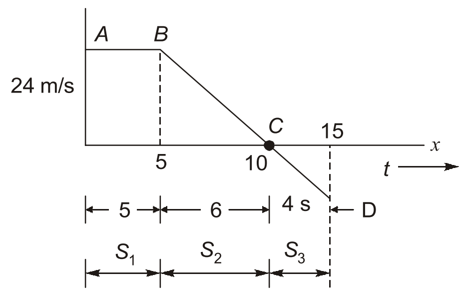


Figure : 2.4

Solution:

(a) **5 seconds :**

$$u = \text{initial velocity} = 24 \text{ m/s constant}$$

$$S_1 = \text{distance covered} = 24 \times 5 = 120 \text{ m}$$

(b) Retardation,

$$a = -4 \text{ m/s}^2$$

$$u - at_2 = 0$$

$$24 - 4 \times t_2 = 0$$

$$t_2 = 6 \text{ second}$$

Velocity becomes zero.

Distance S_2 upto point C

$$S_2 = ut_2 - \frac{1}{2}at_2^2$$

$$= 24 \times 6 - \frac{1}{2} \times 4 \times 6^2 = 144 - 2 \times 36 = 72 \text{ m}$$

At point C, velocity = 0, particle starts moving in reverse direction,

$$t_3 = 4$$

and

$$S_3 = -\frac{1}{2}a \times t_3^2 = -\frac{1}{2} \times 4 \times 4^2 = -32 \text{ m}$$

$$\text{Total distance covered} = 120 + 72 - 32 = 160 \text{ m}$$

Practice Q.2.2 A particle moving along x-axis, with an initial velocity, 20 m/s. The velocity remains constant for first five seconds, thereafter an acceleration of –3 m/s^2 acts on the particle. Determine when the velocity of the particle becomes zero and distance travelled by the particle in 20 seconds.

Ans. [t = 11.67 seconds velocity becomes zero, $S = S_1 - S_2 = 100 - 37.5 = 62.5$ m]

Question 2.3 A particle moves along a straight line such that its displacement is $S = 7t^2 + 10t + 3$ where S is in metre and t in seconds. Determine displacement, velocity and acceleration at $t = 0$, $t = 10$ seconds.

Solution:

Displacement, $S = 7t^2 + 10t + 3$ m

at $t = 0$, $S = 3$ m ...(1)

at $t = 10$ s, $S = 7 \times 100 + 10 \times 10 + 3 = 700 + 100 + 3 = 803$ m ...(2)

Velocity, $V = \dfrac{ds}{dt} = 14t + 10$ m/s

at $t = 0$, $V = 10$ m/s ...(3)

$t = 10$ s, $V = 140 + 10 = 150$ m/s ...(4)

Acceleration, $a = \dfrac{dV}{dt} = 14$ m/s^2 (constant)

at $t = 0$, $t = 10$ s, acceleration, a = 14 m/s^2

Practice Q.2.3 Displacement of a particle is given by $S = 2t^2 - 20t + 40$, where S is in metre and t in seconds. Determine the displacement, velocity and acceleration at $t = 0$, $t = 12$ s.

Ans. [40 m, 88m, –20 m/s, 28 m/s, 4 m/s^2 constant]

Question 2.4 A lift moves up with a constant acceleration a_1, upto a height of 240 m and then 120 m with a constant retardation and then comes to rest. Total travel time is 30 seconds. Determine (a) acceleration; (b) retardation; (c) maximum velocity, if acceleation is half the retardation.

Solution:

Figure 2.5 shows how the velocity goes to maximum and the to zero.

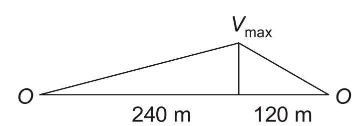


Figure : 2.5

$$\text{Average velocity throughout} = \frac{V_{max}}{2}$$

$$\text{Time taken} = 30 \text{ seconds}$$

$$\text{Total distance travelled} = 240 + 120 = 360 \text{ m}$$

So, $$\frac{V_{max}}{2} = \frac{360}{30} = 12 \text{ m/s}$$

$$V_{max} = 24 \text{ m/s} \quad ...(1)$$

Acceleration, $$a_1 = -\frac{1}{2}a_2 \text{ (retardation)}$$

$$\frac{V_{max}}{a_1} + \frac{V_{max}}{a_2} = 30$$

$$a_2 = -2a_1$$

$$\frac{24}{a_1} + \frac{24}{2a_1} = 30$$

$$\frac{36}{a_1} = 30$$

Acceleration, $$a_1 = \frac{36}{30} = +1.2 \text{ m/s}^2$$

Retardation, $$a_2 = -2.4 \text{ m/s}^2$$